E 5267 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2018

Fifth Semester

Core Course—MATHEMATICAL ANALYSIS

(Common for Model I and Model II B.Sc. Mathematics and B.Sc. Computer Applications)

[2013 Admission onwards]

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions from this part.*
Each question carries 1 *mark.*

1. Define the supremum of a non-empty subset of R. Find the supremum of the set $\left\{1+\frac{(-1)^n}{n}, n \in N\right\}$.
2. If $S=\left\{\frac{1}{n}-\frac{1}{m}, n,m \in N\right\}$. Find the infimum and supremum of S.
3. Define a dense set. Give an example of a set which is dense in itself.
4. Show that the set $S=\left\{1,-1,\frac{1}{2},\frac{-1}{2},\frac{1}{3},\frac{-1}{3}\ldots\right\}$ is neither open nor closed .
5. Define limit point of a set. What is the closure of the set Q, the set of rational numbers.
6. Define the term convergence of a sequence of real numbers. Give an example of a convergent sequence.
7. Find $\lim \frac{2n^2-5}{3n^2+7n}$.
8. Give an example of a sequence which oscillates infinitely.
9. Find the modulus and argument of $z=-1-i\sqrt{3}$.
10. State De-Moivre's formula.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Show that there is no rational number whose square is 2.
12. State Dedekind's form of completeness property.

Turn over

13. Show that union of two closed sets is closed.

14. Is union of an arbitrary collection of closed sets closed. Justify your answer.

15. Give an example of a bounded set having :

    (i) no limit point.

    (ii) infinite number of limit points.

16. Consider the sequence $\{S_n\}$ where $S_n = \frac{(-1)^{n-1}}{n!}, n \in N$. Is the sequence bounded. Is the sequence convergent. If so find the limit.

17. Show that if $\lim a_n = a$, then $\lim |a_n| = |a|$. Is the converse True. Justify.

18. Give examples of two sequences whose sum, product and quotient converges but the sequences are not convergent.

19. Show that the sequence $\{S_n\}$ where $S_n = 1 + \frac{1}{2} + ... + \frac{1}{n}$ cannot converge.

20. Show that Im (iz) = Re (z).

21. Find all roots of $(2i)^{1/2}$ in rectangular co-ordinates.

22. Use de Moivre's formula to derive $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. State the properties of R that makes it into a complete-ordered field.

24. Show that union of an arbitrary family of open sets is open.

25. Show that a countable union of countable sets is countable.

26. Show that derived set of a set is closed.

27. Prove that a sequence cannot converge to more than one limit.

28. Show that $\lim_{n\to\infty} \frac{1}{n}\left[1 + 2^{1/2} + 3^{1/3} + .. + n^{1/n}\right] = 1$.

29. Show that $\{S_n\}$, where $S_n = \left(1+\frac{1}{n}\right)^n$ is convergent and that $\lim\left(1+\frac{1}{n}\right)^n$ lies between 2 and 3.

30. Locate the numbers $Z_1 + Z_2$ and $Z_1 - Z_2$ vectorially when $Z_1 = 2i$, $Z_2 = \frac{2}{3} - i$.

31. Prove that $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$ and $|Z_1 - Z_2| \geq ||Z_1| - |Z_2||$.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Prove that the set of rational numbers is not order complete.

    (b) If a and b are any two positive real numbers, prove that there exists a positive integer n such that $na > b$.

33. (a) Prove that subset of a countable set is countable.

    (b) State and prove Cauchy's general principle of convergence.

34. (a) State and prove Cauchy's first theorem on limits.

    (b) Is the converse true. Justify.

35. (a) Show that the sequence $\{S_n\}$ defined by $S_{n+1} = \sqrt{3\ S_n}$, $S_1 = 1$ converges to 3.

    (b) Show that the sequence $\{b_n\}$ where $b_n = \left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + .. + \frac{1}{(2n)^2}\right]$ converges to zero.

    (c) Show that for any number x, $\lim \frac{x}{n!} = 0$.

$(2 \times 15 = 30)$